An accessible microwave cavity experiment for plasma density determination

J. Oliveira[1], D. Hachmeister[1], P. D. Lourenço[1], J. Brotankova[2], H. Fernandes[1]

[1]Instituto de Plasmas e Fusão Nuclear, Universidade de Lisboa, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

[2]Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University, Břehová 7, 115 19 Prague 1, Czech Republic

E-mail: `joao.s.oliveira@tecnico.ulisboa.pt`

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Abstract. Plasmas are indissociable from microwave science. Effectively, the most fundamental properties of any plasma arise from its plasma frequency. However, combining plasma physics with electromagnetic (EM) wave propagation is challenging for physics students. An electromagnetic cavity poses an excellent opportunity to understand the behavior of EM standing waves, the issues related to transmitted and reflected power in waveguides, coupling, and how free charges in the plasma affect its resonant frequency and quality factor, interlinking the properties of matter with wave propagation. The inclusion of an external magnetic field allows students to explore the underlying principles of magnetic confinement in a linear geometry.

1. Introduction

Measuring the shift of the resonant frequency in a microwave cavity can be used to determine the electromagnetic properties of material samples within the cavity [1]. If a low-temperature plasma fills the cavity, the resonant frequency shift will depend on the plasma's electric permittivity, which in turn depends on its electron density [2]. This technique can measure electron densities between roughly 10^{16} and 10^{21} m^{-3} depending on the frequency resolution of the experiment. This type of density-measurement diagnostic has applications in the area of material processing plasmas [3, 4], and in low-temperature atmospheric pressure plasma science [5].

A microwave cavity refers to a volume enclosed by a conducting surface which can store electromagnetic (EM) energy. It can be thought of as the microwave analog of an LC circuit, albeit with multiple resonance frequencies and much larger quality factors. Three main processes govern the energy losses in a cavity: conduction losses in the cavity walls, conduction loss in the dielectric material filling the cavity, and losses through access ports or holes in the conducting surface. In the first-order approximation, these losses cause the same resonance peak broadening as the resistive losses in an RLC circuit, causing both the resonance frequency shift and a decrease in the quality factor.

The aim of this paper is to present an accessible (under 3000$) plasma experiment that is easily replicated. We believe that this experiment is well suited for providing undergraduate students with hands-on experience on basic plasma physics experimental techniques.

1.1. Cylindrical microwave cavity

Two types of resonant modes occur in electromagnetic cavities: transverse-electric modes (TE) and transverse-magnetic modes (TM). In the TE (TM) modes, the electric (magnetic) field lines are transverse to the longitudinal direction.

A comprehensive derivation of all the resonant modes of a cavity is available in chapter 6 of the book by David Pozar [6]. In a cylindrical cavity filled by a homogeneous and isotropic medium, the resonant frequencies of the TM modes are:

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}, \tag{1}$$

where the indices n, m, l correspond to the number of maxima of the EM fields in the azimuthal (ϕ), radial (ρ), and axial (z) directions, c is the speed of light in vacuum, μ_r and ϵ_r are the relative permeability and permittivity of the medium inside the cavity, p_{nm} is the m^{th} zero of the n^{th} order Bessel function of the first kind $J_n(x)$ (see table 1), d is the cavity's length, and a its radius.

m \ n	0	1	2	3	4	5
1	2.4048	3.8317	5.1356	6.3801	7.5883	8.7714
2	5.5200	7.0155	8.4172	9.7610	11.064	12.338
3	8.6537	10.173	11.619	13.015	14.372	15.700
4	11.791	13.323	14.795	16.223	17.615	18.980
5	14.930	16.470	17.959	19.409	20.826	22.217
6	18.071	19.615	21.116	22.582	24.019	25.430

Table 1. p_{nm}, m^{th} zero of the n^{th} order Bessel function of the first kind, for varying values of m and n.

The TM_{010} mode is the fundamental TM mode and has some remarkable properties. The resonant frequency of this mode does not depend on the cavity's length - see (1). Furthermore, this mode's electric field is parallel to the axial direction, while the magnetic field is parallel to the azimuthal direction. Because of this, it is easily excitable by a loop antenna with its axis in the azimuthal direction (see figure 1).

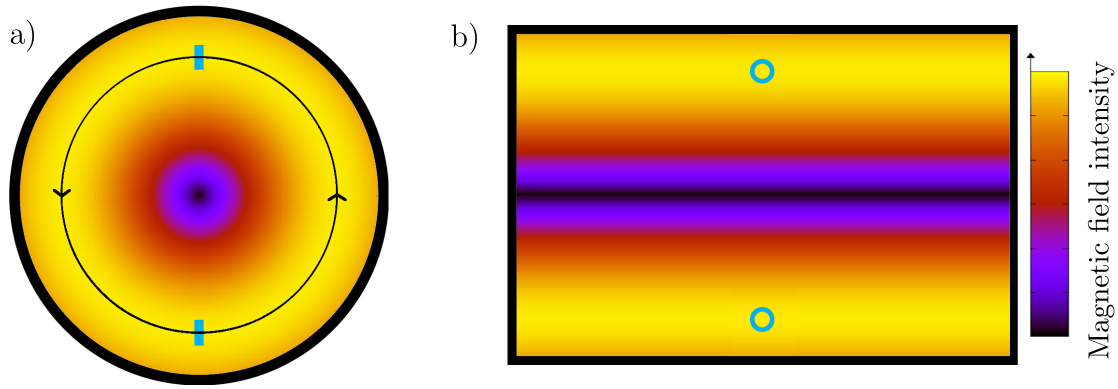

Figure 1. Transverse (a) and side (b) views of the magnetic field intensity of the TM_{010} mode in a cylindrical cavity. The electric field is axial and the magnetic field is azimuthal. The configuration of the loop antennas used to measure the transmittance of the cavity is in blue. The antennas are placed near the region of maximum intensity of the magnetic field; the black line in a) indicates this region.

1.2. Plasma permittivity

If a homogeneous and isotropic plasma fills the cylindrical cavity with an electron collision frequency much smaller than the cavity's resonance frequency and a Maxwellian electron distribution, the relative permittivity of the plasma is:

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}, \tag{2}$$

where w_p is the plasma frequency

$$\omega_p^2 = \frac{e^2 n_e}{\epsilon_0 m_e} \ , \tag{3}$$

n_e is the electron density, e is the electron charge, m_e the electron mass, and w the angular frequency of the electric field.

Substituting (2) in (1) and assuming that the resonance frequency shift Δf caused by the plasma is small with respect to the vacuum resonance frequency f_{010}, the plasma electron density with respect to Δf is given by:

$$n_e = \frac{8\pi^2 m_e \epsilon_0}{e^2} f_{010} \Delta f. \tag{4}$$

If an external magnetic field is present, (4) is only valid as long as the electric field of the mode is parallel to the external magnetic field. This approximation further assumes that the plasma does not change the shape of the mode's electric field.

2. Experimental Setup

2.1. Discharge chamber

The schematic in figure 2 depicts the resonant cavity used in this experiment. It consists of a copper cylinder with a 64 mm diameter and 50 mm length. The cylinder bases

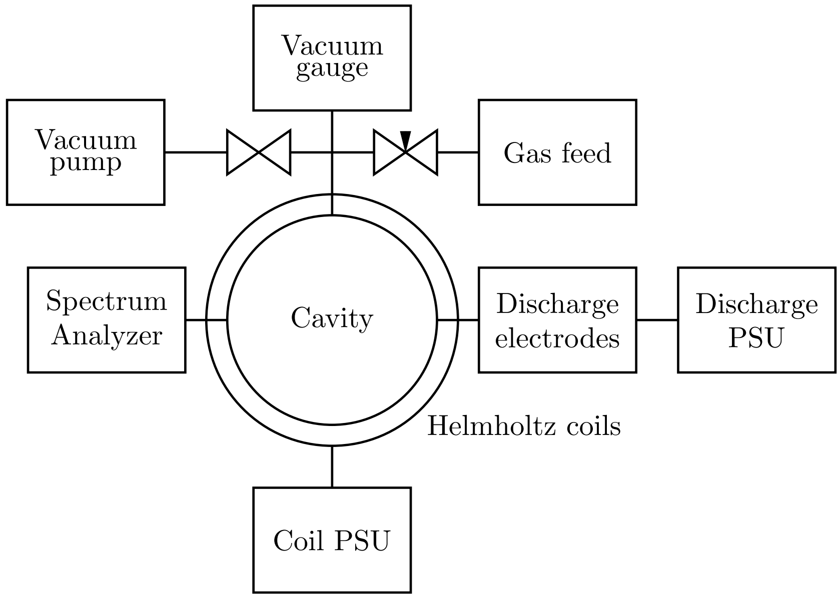

Figure 2. Block diagram of the experimental setup. The coils PSU can deliver up to 14 A (20 mT). The vacuum system combined with the gas injection allows for the pressure range from 1 to 300 Pa. The spectrum analyzer also acts as the microwave source for measuring the transmittance of the resonant cavity.

a) b)

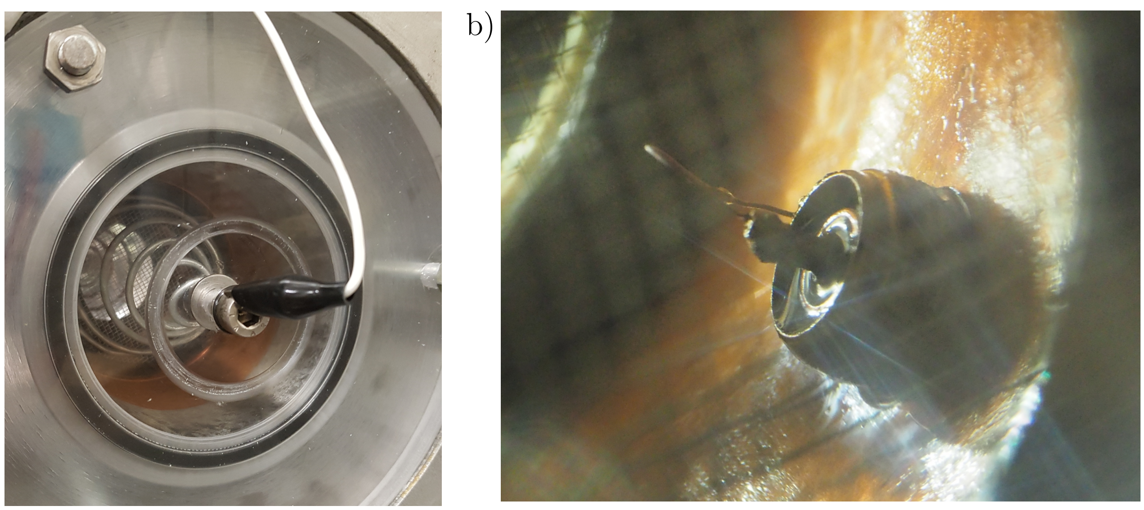

Figure 3. a) Cavity top view with the power-source connected to the spring which presses on the aluminum grid. b) Loop antenna used to excite the cavity as illustrated in figure 1. The antenna is soldered to a BNC connector.

consist of an aluminum mesh with a spacing inferior to one-tenth of the microwave wavelength used in the experiment. A spring bolted to the opposite side to the vacuum window holds the mesh in place, as shown in figure 3 a). The whole cavity lies between two Helmholtz coils that generate a magnetic field of 1.4 mT per Ampere, up to the maximum current of 14 A.

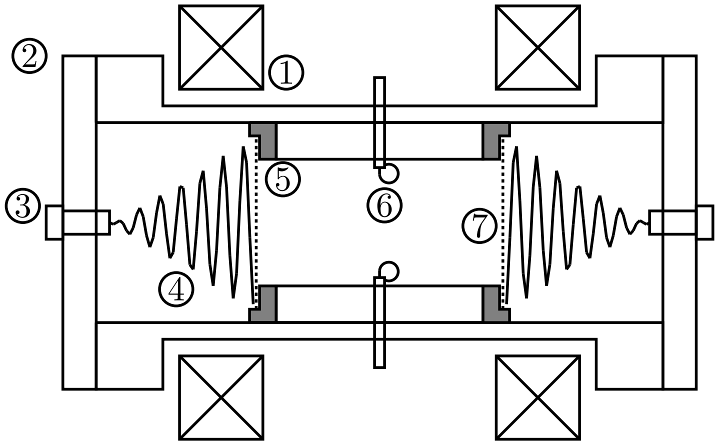

Figure 4. Side view diagram of the resonant cavity setup. 1 - Helmholtz coils; 2 - Acrylic window; 3 - Discharge electrode; 4 - Coil spring; 5 - Mesh insulator; 6 - Antenna; 7 - Aluminum mesh

2.2. Vacuum and gas injection

The chamber is kept under vacuum by a rotary pump that can reach pressures of the order of 1 Pa. The pump is a low-cost model that is not designed for continuous operation. Since the experiment does not require continuous operation this is a good compromise that lowers the overall cost. The working gas flows into the chamber through a needle valve. It is possible to select a working pressure where the vacuum pump balances the gas inflow by adjusting the valve. This setup uses two different working gases: helium and argon.

2.3. The Penning discharge

A Cold Cathode Fluorescent Light (CCFL) inverter serves as a current source to generate a Penning discharge inside the cavity [7]. The inverter converts a 12 V DC input to 1 kV AC output connected to both the electrodes inside the cavity. The aluminum meshes that work as electrodes are electrically insulated from the cylinder's lateral surface (see figure 4). This configuration ensures that the discharge permeates the entire cavity uniformly. The CCFL inverter managed to sustain discharges with working gas pressures between 10 and 300 Pa.

2.4. Measurements of the resonance

Two loop antennas lie in the middle of the cylinder wall opposite each other. The antennas have a 4 mm radius with their axis coincident with the magnetic field line of highest intensity of the TM_{010} mode, see figure 1 a).

One of the antennas injects the microwaves in the cavity, and the other antenna receives them. The highest observed resonance corresponded to a frequency of 3560 MHz (see figure 5). This frequency is slightly below the theoretical TM_{010} mode frequency

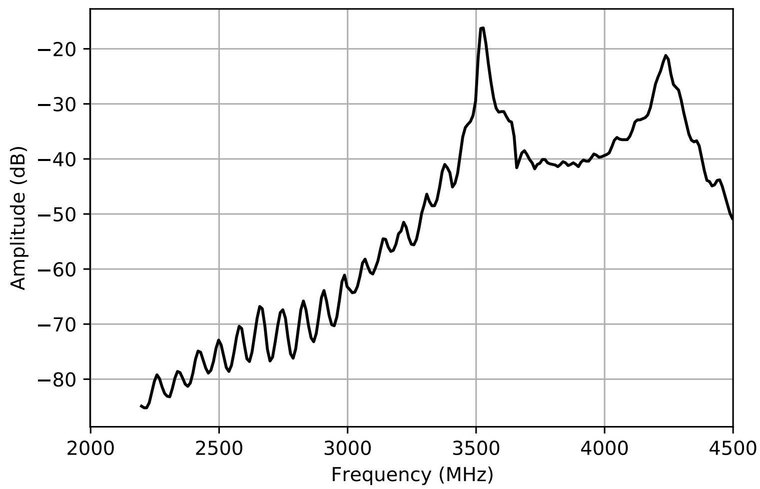

Figure 5. Transmitted power collected on the receiving loop as a function of frequency. The first resonant peak is clearly visible at 3.55 GHz with the chamber filled by a helium plasma at 45 Pa background pressure.

(3586 MHz). This difference could stem from a miscalibration of the spectrum analyzer or the cavity's nonideal shape (some access ports on the cavity).

2.5. Spectrum analyzer

To generate and receive the microwaves passing through the cavity, we used the inexpensive ARNIST SSA-TG R2 spectrum analyzer. The device has the frequency range from 35 to 4500 MHz with the resolution of 200 kHz. It can also be connected to a PC through a USB interface to easily store spectral data for later analysis. The analyzer sweeps the frequency spectrum measuring the attenuation (in dBm) for each frequency. The spectrum analyzer can run continuously, sweeping the spectrum each second, allowing the online visualization of the changes in resonant frequency.

3. Experimental Procedure

The cavity is pumped down for about one hour until a stable pressure is reached ($\approx$ 1 Pa). The spectrum analyzer then performs a frequency sweep to determine the resonance frequency of the TM_{010} mode used as the reference to calculate the frequency shift.

Opening the gas valve fills the cavity with gas, and the CCFL inverter rapidly starts a Penning discharge, forming a plasma. The frequency shift of the TM_{010} becomes immediately evident on the spectrum analyzer's display.

The needle valve controls the gas pressure inside the cavity to measure the spectral attenuation for different working pressures. The power supply connected to the

Helmholtz coils can also be adjusted up to 14 A, providing a two-dimensional parameter space.

The data retrieved from the spectrum analyzer can be post-processed to determine the frequency shift accurately. In our experiments, we found that fitting a parabola to the top of the attenuation curve's peak provides a reasonable estimate of the frequency shift.

4. Results

When a plasma forms inside the chamber, the resonant frequency shifts by an amount proportional to the average plasma density. Figure 6 shows the spectral analyzer's response in the absence of plasma and with an argon plasma at 30 Pa.

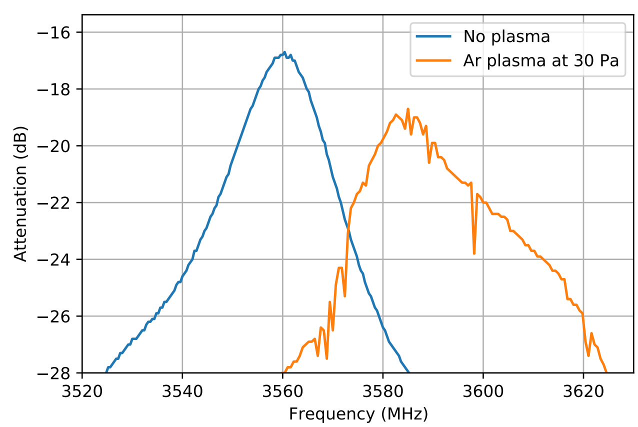

Figure 6. Detail of the transmitted power spectrum for an argon plasma at 30 Pa (orange) and without plasma (blue) - discharge switched off.

4.1. Density vs. pressure

We observed that the electron density is a non-monotonic function of pressure in conformity with previous observations [3, 4]. Figure 7 shows the inferred density as a function of pressure in the chamber for two types of working gases: helium and argon. Both the gases achieve roughly the same maximum electron density at different pressures.

4.2. Effect of an external magnetic field

Adding an external magnetic field parallel to the cavity axis causes a decrease in the average electron density, as can be seen in figure 8.

Figure 9 shows the electron density as a function of pressure for different values of the magnetic field. A decrease is observed in the region before the peak in electron density and disappears at pressures above 200 Pa.

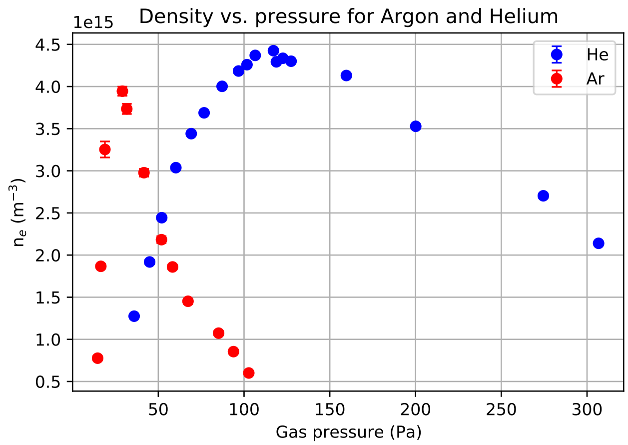

Figure 7. Electron density as a function of background pressure for both helium and argon plasmas.

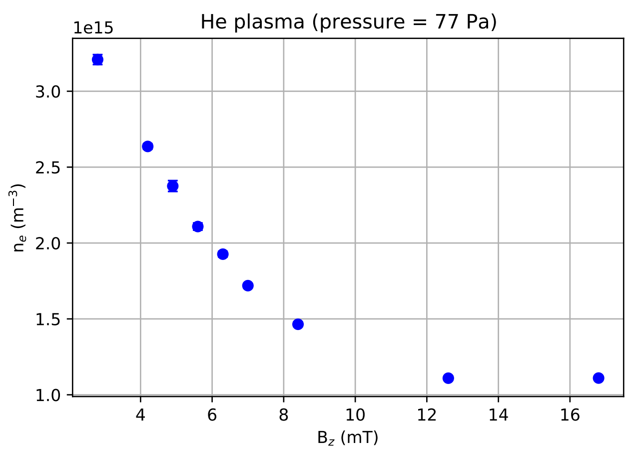

Figure 8. Electron density as a function of magnetic field for a helium plasma at the background pressure of 77 Pa.

5. Discussion

The first observation is that a plasma-filled cavity leads to an increase in the resonant frequency. This observation implies a wavelength increase inside the cavity. Thus, the plasma effectively decreases the refractive index of the medium.

The second observation is that there are two forces at play that govern the ionization processes. With an initial increase in pressure, the frequency of penning ionization increases. After reaching a peak, electron density decreases with a further increase in the pressure. This density drop happens because the increased number of neutrals in the cavity can dissipate more energy, and the discharge power is constant.

Finally, an external magnetic field's presence decreases the frequency shift of the TM_{010} mode. Since the mode's electric field is parallel to the external magnetic field, it cannot directly affect the frequency shift. The remaining explanation is that the

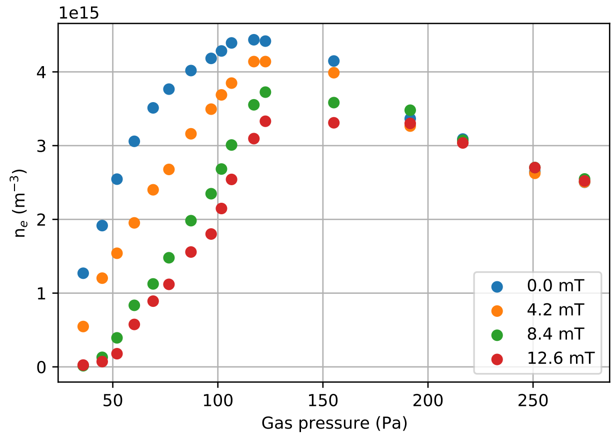

Figure 9. Electron densities for a helium plasma as a function of background pressure for different magnetic fields.

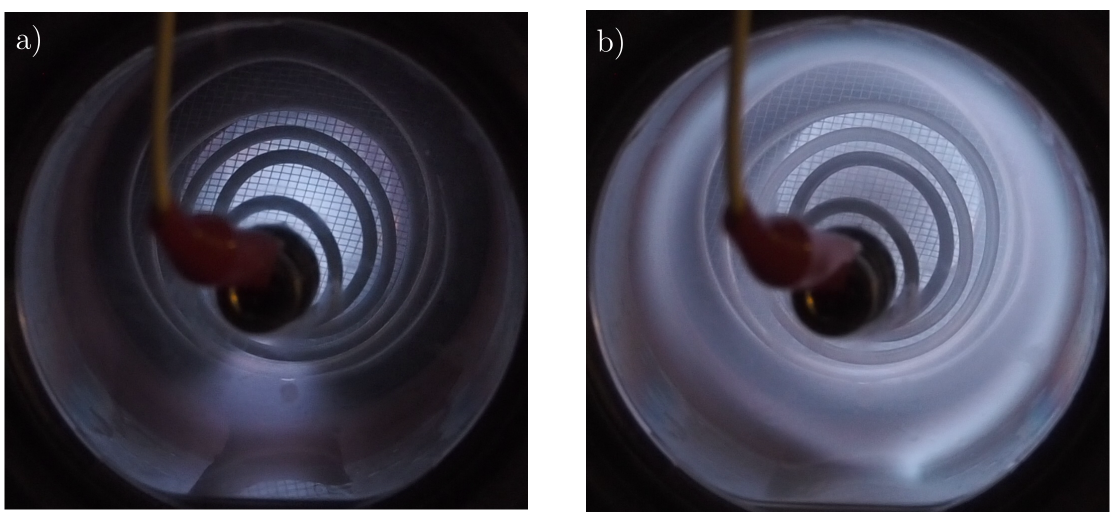

Figure 10. Plasma snapshot showing the plasma inside the cavity with: a) no external magnetic field; b) external field of ≈ 20 mT. In the presence of the magnetic field, the plasma is pinched in the center of the cavity and expands outwards crossing the metallic grid and coming outside of the electromagnetic cavity to the magnetic confinement.

external magnetic field decreases the average electron density by decreasing the density everywhere or changing the electrons' spatial distribution. The addition of an external magnetic field can generate a compression of the plasma quantified by the normalized pressure - the plasma pressure divided by the magnetic pressure ($B^2/2\mu_0$). The plasma will contract due to the magnetic pressure. This phenomenon is easily observable in a transparent glow discharge tube, for example [8]. In our experiment, we observed that the magnetic field forces the plasma out of the electrodes' boundary, as can be seen in figure 10.

This phenomenon is called a θ-pinch and is the central concept in magnetic confinement fusion devices [9]. For higher gas pressures we observed no changes in density with the magnetic field.

6. Conclusions

We developed an inexpensive plasma experiment to educate undergraduate students on electromagnetic cavity's behavior under different plasma parameters. The main goal was to give students experience in the application of microwave technology to plasma science.

We consider that the EM cavity is an excellent introductory tool to both EM waves - by studying the resonance phenomenon - and the physics of ionized gases - by understanding the plasma's refractive index and how magnetic confinement works. Furthermore, it allows the student to glimpse at the intricate relationship between plasma and microwaves.

The current setup allows the students to explore the relationship between electron density, gas pressure, and magnetic field by giving them hands-on training to manually control and see the plasma and the resulting frequency shift.

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